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A classical treatment of the long-range radiative interaction of small particles

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Abstract. The induced oscillations of small particles in a massless classical field (acoustic, electromagnetic etc) result in the appearance of the time-averaged long-range radiative forces. These forces are proportional to the square of the field amplitude and inversely proportional to the distance between the particles. The radiative forces predominate at large distances. Their structure is universal for arbitrary classical fields.

1. Introduction

Recently it was found that liquid compressibility leads to the appearance of two types of long-range radiative forces between small particles (gas bubbles and solid corpuscles) oscillating in a sound field [1-3]. One type is the mutual attraction or repulsion of particles for which the forces are directed along $\boldsymbol{l}$, where $\boldsymbol{l}=\boldsymbol{r}_2-\boldsymbol{r}_1$, $\boldsymbol{r}_{1,2}$ being the position vectors of particle centres. The second type is the transfer and the mutual rotation of the particles for which the forces are directed along $\boldsymbol{k}$, where $\boldsymbol{k}$ is a wavevector. Both these types of time-averaged forces are proportional to the square of the sound wave amplitude and inversely proportional to the distance between the particles. The relative motion of two particles under the action of the long-range radiative forces and short-range Bjerknes forces was investigated in [4].

It is interesting to transfer the corresponding results to the problem of the radiative long-range interaction between two small particles due to an arbitrary classical field. It is shown in this paper that the induced dipole oscillations of the charged particles in an electromagnetic field result in the appearance of analogous radiative forces. These forces are caused by secondary radiation of the charged particles; at large distances they may predominate over the Coulomb forces. This even leads to the possibility of forming bound states between two like charges. The corresponding values of field intensity are obtained. In this paper the radiative interaction of two non-relativistic particles through a scalar field is also investigated.

Such possible interdisciplinary transfer between acoustics, electrodynamics and the classical theory of field is based on the formal analogy of corresponding equations. It is not new in physics, having already been considered in, for example, [5-7]. It is also clear that the term particle itself does not have the same significance in fluid mechanics that it has in electrodynamics. Many authors have recently investigated extended models of electrons [8-13]. In this paper the charged particles are considered to be the usual classical particles without internal structure.

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2. The radiative interaction of gas bubbles in a liquid under the action of sound waves

Let us consider the bubble monopole oscillations under the action of sound waves $P_{ac} = A\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$, where A is the pressure amplitude and ω is the cyclic frequency. These oscillations give rise to the scattering field

$$\varphi_s = \varphi_{s1} + \varphi_{s2} = -\frac{R_1^2\Delta\dot{R}_1(t-|\boldsymbol{r}-\boldsymbol{r}_1|/c)}{|\boldsymbol{r}-\boldsymbol{r}_1|} - \frac{R_2^2\Delta\dot{R}_2(t-|\boldsymbol{r}-\boldsymbol{r}_2|/c)}{|\boldsymbol{r}-\boldsymbol{r}_2|} \tag{2.1}$$

where $\varphi_{s1,2}$ are the delayed velocity potentials created by the first and second bubble respectively, $R_{1,2}$ are the mean radii, $\Delta R_{1,2}$ are their shifts, $\boldsymbol{r}_{1,2}$ are the position vectors of the mean positions of the bubble centres and c is the speed of sound in the liquid.

The time dependences of the bubble radii can be represented by the expressions

$$\Delta R_{1,2} = a_{1,2}\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{1,2} - \chi_{1,2}). \tag{2.2}$$

The time-averaged force acting, for example, on the first bubble is given by [14]

$$\boldsymbol{F}_1 = -4\pi R_1^2\langle\Delta R_1 \nabla P\rangle. \tag{2.3}$$

The resulting pressure is $P = P_{ac} + P_{s2}$ where $P_{s2} = -\rho\dot{\varphi}_{s2}$ where ρ is the density of the liquid.

It should be noted that in [1] only the pressure P_{s2} has been taken into account. This leads to the loss of the second type of long-range forces mentioned in section 1.

In order to calculate the amplitudes $a_{1,2}$ and phases $\chi_{1,2}$, we used the equations of the radius pulsations [14]

$$\begin{aligned}
\Delta\ddot{R}_1 + \omega_1^2\Delta R_1 + \omega\delta_1\Delta\dot{R}_1 &= -\frac{A\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_1)}{\rho R_1} - \frac{R_2^2}{R_1 l}\Delta\ddot{R}_2(t - l/c) \\
\Delta\ddot{R}_2 + \omega_2^2\Delta R_2 + \omega\delta_2\Delta\dot{R}_2 &= -\frac{A\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_2)}{\rho R_2} - \frac{R_1^2}{R_2 l}\Delta\ddot{R}_1(t - l/c).
\end{aligned} \tag{2.4}$$

The equations (2.4) take account of the additional delayed pressures [2, 3], l is the distance between the two gas bubbles, $\delta_{1,2}$ are the absorption constants, and $\omega_{1,2}$ are the resonance frequencies of gas bubbles.

The substitution of (2.2) into (2.4) leads to the nonlinear algebraic system for $a_{1,2}$ and $\chi_{1,2}$. To first order in $R_{1,2}/l$, the solution is given by

$$\begin{aligned}
a_1 &= a_{10} + a_{11} = -\frac{A}{\rho\omega^2 R_1[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}}\left(1 + \frac{R_2}{l}\frac{\cos(kl + \boldsymbol{k}\cdot\boldsymbol{l})}{[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}}\right) \\
a_2 &= a_{20} + a_{21} = -\frac{A}{\rho\omega^2 R_2[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}}\left(1 + \frac{R_1}{l}\frac{\cos(kl - \boldsymbol{k}\cdot\boldsymbol{l})}{[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}}\right) \\
\chi_1 &= \chi_{10} + \chi_{11} = \tan^{-1}\left(\frac{\delta_1}{\omega_1^2/\omega^2-1}\right) + \frac{R_2}{l}\frac{\sin(kl + \boldsymbol{k}\cdot\boldsymbol{l})}{[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}} \\
\chi_2 &= \chi_{20} + \chi_{21} = \tan^{-1}\left(\frac{\delta_2}{\omega_2^2/\omega^2-1}\right) + \frac{R_1}{l}\frac{\sin(kl - \boldsymbol{k}\cdot\boldsymbol{l})}{[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}}
\end{aligned} \tag{2.5}$$

where a_{10}, a_{20}, χ_{10} and χ_{20} are the corresponding values at the limit $R_{1,2}/l \to 0$.

The substitution of (2.5) into (2.3) allows us to obtain the following expressions for the radiative forces to first order in $kR_{1,2} \ll 1$:

$$\boldsymbol{F}_1 = \boldsymbol{F}_{1l} + \boldsymbol{F}_{1k} \qquad \boldsymbol{F}_2 = \boldsymbol{F}_{2l} + \boldsymbol{F}_{2k} \tag{2.6}$$

where the forces of the first type (in the direction of $\boldsymbol{l}$) are

$$\boldsymbol{F}_{1l}=\frac{2\pi R_1 R_2 A^2}{\rho\omega^2[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}}\frac{\boldsymbol{l}}{l^3}$$
$$\times[\cos(kl+\boldsymbol{k}\cdot\boldsymbol{l}+\chi_{20}-\chi_{10})+kl\sin(kl+\boldsymbol{k}\cdot\boldsymbol{l}+\chi_{20}-\chi_{10})] \tag{2.7}$$

$$\boldsymbol{F}_{2l}=-\frac{2\pi R_1 R_2 A^2}{\rho\omega^2[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}}\frac{\boldsymbol{l}}{l^3}$$
$$\times[\cos(kl-\boldsymbol{k}\cdot\boldsymbol{l}+\chi_{20}-\chi_{10})+kl\sin(kl-\boldsymbol{k}\cdot\boldsymbol{l}+\chi_{20}-\chi_{10})] \tag{2.8}$$

and the forces of the second type (directed along $\boldsymbol{k}$) are

$$\boldsymbol{F}_{1k}=\frac{2\pi R_1 A^2\delta_1\boldsymbol{k}}{\rho\omega^2[(\omega_1^2/\omega^2-1)^2+\delta_1^2]}+\frac{2\pi R_1 R_2 A^2\boldsymbol{k}\sin(kl+\boldsymbol{k}\cdot\boldsymbol{l}+\chi_{10})}{\rho\omega^2 l[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}} \tag{2.9}$$

$$\boldsymbol{F}_{2k}=\frac{2\pi R_2 A^2\delta_2\boldsymbol{k}}{\rho\omega^2[(\omega_2^2/\omega^2-1)^2+\delta_2^2]}+\frac{2\pi R_1 R_2 A^2\boldsymbol{k}\sin(kl-\boldsymbol{k}\cdot\boldsymbol{l}+\chi_{20})}{\rho\omega^2 l[(\omega_1^2/\omega^2-1)^2+\delta_1^2]^{1/2}[(\omega_2^2/\omega^2-1)^2+\delta_2^2]^{1/2}}. \tag{2.10}$$

The first terms in (2.9) and (2.10) are the usual forces of radiative pressure [14–18]. In the limit $kl \ll 1$ (incompressible liquid) the forces (2.7) and (2.8) transform to the well known short-range Bjerknes forces [14–16]. The radiative forces of the second type which depend on the distance $\boldsymbol{l}$ between the particles are long range. It follows from (2.9) and (2.10) that the sum of these forces and the forces (2.7) and (2.8) is not equal to zero. To our knowledge, this result was first found in [1]. The sum $\boldsymbol{F}_1+\boldsymbol{F}_2$ is equal to impulse taken away by the scattering sound wave [1].

3. The radiative interaction of the charges in the electromagnetic wave

It is interesting to transfer the corresponding results into the problem of the radiative interaction of two spinless charged particles in the electromagnetic plane wave $\boldsymbol{E}_{\text{ext}}=\boldsymbol{E}_0\cos(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})$ and $\boldsymbol{H}_{\text{ext}}=\boldsymbol{H}_0\cos(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})$ where $\boldsymbol{E}_0$ is the amplitude of the electric intensity vector, and $\boldsymbol{H}_0=[\boldsymbol{k}/k, \boldsymbol{E}_0]$ is the amplitude of the magnetic intensity vector. Let $\boldsymbol{r}_{10}$ and $\boldsymbol{r}_{20}$ be the position vectors of the mean positions of two particles, and let $\boldsymbol{\xi}_{1,2}$ be their displacements. The position vectors of the current positions are $\boldsymbol{r}_1=\boldsymbol{r}_{10}+\boldsymbol{\xi}_1$ and $\boldsymbol{r}_2=\boldsymbol{r}_{20}+\boldsymbol{\xi}_2$. We suppose that $|\boldsymbol{\xi}_{1,2}|\ll l, \lambda$ where $\boldsymbol{l}=\boldsymbol{r}_{20}-\boldsymbol{r}_{10}$ and λ is the wavelength.

Taking into account the electromagnetic field scattering by neighbouring particles, one obtains the following ponderomotive equations:

$$m_1\ddot{\boldsymbol{r}}_1=e_1(\boldsymbol{E}_{\text{ext}}(\boldsymbol{r}_1,t)+\boldsymbol{E}_{r2}(\boldsymbol{r}_1,t))+(e_1/c)[\dot{\boldsymbol{r}}_1,\boldsymbol{H}_{\text{ext}}(\boldsymbol{r}_1,t)+\boldsymbol{H}_{r2}(\boldsymbol{r}_1,t)] \tag{3.1}$$

$$m_2\ddot{\boldsymbol{r}}_2=e_2(\boldsymbol{E}_{\text{ext}}(\boldsymbol{r}_2,t)+\boldsymbol{E}_{r1}(\boldsymbol{r}_2,t))+(e_2/c)[\dot{\boldsymbol{r}}_2,\boldsymbol{H}_{\text{ext}}(\boldsymbol{r}_2,t)+\boldsymbol{H}_{r1}(\boldsymbol{r}_2,t)] \tag{3.2}$$

where $e_{1,2}$ and $m_{1,2}$ are the charges and masses of the particles, c is the velocity of light, and $\boldsymbol{E}_{r1}$, $\boldsymbol{H}_{r1}$($\boldsymbol{E}_{r2}$, $\boldsymbol{H}_{r2}$) are the electric and magnetic intensity vectors of field created by dipole oscillations of the first (second) particle.

In order to calculate the scattering fields we use the well known expressions for the delayed potentials [19]

$$\boldsymbol{E}_{r1,2}=-\frac{e_{1,2}}{c^2R_{1,2}}[\ddot{\boldsymbol{\xi}}_{1,2}(t'_{1,2})-\boldsymbol{n}_{1,2}(\boldsymbol{n}_{1,2}\cdot\ddot{\boldsymbol{\xi}}_{1,2}(t'_{1,2}))] \tag{3.3}$$

$$\boldsymbol{H}_{r1,2}=-\frac{e_{1,2}}{c^2R_{1,2}}[\boldsymbol{n}_{1,2},\ddot{\boldsymbol{\xi}}_{1,2}(t'_{1,2})] \tag{3.4}$$

where $R_{1,2}$ are the propagation distances of radiation and the unit vectors $\boldsymbol{n}_{1,2}$ give the direction of radiation. We shall examine only the long-range limit $kl \gg 1$ because the most interesting results are obtained at this limit. The expressions (3.3) and (3.4) should to be calculated at delayed moments of the time [19]

$$t'_{1,2} = t - |\boldsymbol{r} - \boldsymbol{r}_{1,2}(t'_{1,2})|/c. \tag{3.5}$$

The usual Coulomb term is omitted in equation (3.3).

With an accuracy to first order in the field amplitude it follows from (3.1)-(3.5) that

$$m_1\ddot{\boldsymbol{\xi}}_1(t) = e_1\boldsymbol{E}_0\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{10}) - \frac{e_1e_2}{c^2 l}[\ddot{\boldsymbol{\xi}}_2(t-l/c) - \boldsymbol{n}(\boldsymbol{n}\cdot\ddot{\boldsymbol{\xi}}_2(t-l/c))] \tag{3.6}$$

$$m_2\ddot{\boldsymbol{\xi}}_2(t) = e_2\boldsymbol{E}_0\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{20}) - \frac{e_1e_2}{c^2 l}[\ddot{\boldsymbol{\xi}}_1(t-l/c) - \boldsymbol{n}(\boldsymbol{n}\cdot\ddot{\boldsymbol{\xi}}_1(t-l/c))] \tag{3.7}$$

where $\boldsymbol{n} = \boldsymbol{l}/l$.

The solution of this system can be represented as $\boldsymbol{\xi}_1 = \boldsymbol{\xi}_{10} + \boldsymbol{\xi}_{11}$, $\boldsymbol{\xi}_2 = \boldsymbol{\xi}_{20} + \boldsymbol{\xi}_{21}$, where

$$\boldsymbol{\xi}_{10}(t) = \frac{e_1\boldsymbol{E}_0\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{10})}{m_1\omega^2}$$

$$\boldsymbol{\xi}_{20}(t) = -\frac{e_2\boldsymbol{E}_0\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{20})}{m_2\omega^2} \tag{3.8}$$

$$\boldsymbol{\xi}_{11}(t) \approx \frac{e_1e_2^2\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{20} - kl)}{m_1m_2\omega^2c^2l}(\boldsymbol{E}_0 - \boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{E}_0)) \tag{3.9}$$

$$\boldsymbol{\xi}_{21}(t) \approx \frac{e_2e_1^2\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_{10} - kl)}{m_1m_2\omega^2c^2l}(\boldsymbol{E} - \boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{E}_0)). \tag{3.10}$$

In order to calculate the radiative forces, let us consider the displacements $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ as generalised coordinates. Then the position vectors $\boldsymbol{r}_{10}$ and $\boldsymbol{r}_{20}$ are the parameters of our system. The interaction Hamiltonian for the interactions of the first particle with the electromagnetic field is [19]

$$H_1 = e_1\varphi_1(\boldsymbol{r}_{10} + \boldsymbol{\xi}_1) - \frac{e_1\dot{\boldsymbol{\xi}}_1\cdot\boldsymbol{A}_1(\boldsymbol{r}_{10}+\boldsymbol{\xi}_1)}{c} \approx e_1\varphi_1(\boldsymbol{r}_{10}) + e_1\frac{\partial\varphi_1(\boldsymbol{r}_{10})}{\partial\boldsymbol{r}_{10}}\cdot\boldsymbol{\xi}_1 - \frac{e_1\dot{\boldsymbol{\xi}}_1\cdot\boldsymbol{A}_1(\boldsymbol{r}_{10})}{c} \tag{3.11}$$

where φ_1 and $\boldsymbol{A}_1$ are the resulting scalar and vector potentials (which are the sum of the corresponding potentials of the external wave and the wave scattered by the second particle).

The time-averaged force acting on the first particle is [19] (the Coulomb force is omitted)

$$\boldsymbol{F}_1 = -\left\langle\frac{\partial H_1}{\partial\boldsymbol{r}_{10}}\right\rangle = e_1\left\langle\boldsymbol{\xi}_1(t)\cdot\frac{\partial\boldsymbol{E}_1(\boldsymbol{r}_{10},t)}{\partial\boldsymbol{r}_{10}}\right\rangle \tag{3.12}$$

where

$$\boldsymbol{E}_1(\boldsymbol{r}_{10},t) = -\frac{\partial\varphi_1(\boldsymbol{r}_{10},t)}{\partial\boldsymbol{r}_{10}} - \frac{1}{c}\dot{\boldsymbol{A}}_1(\boldsymbol{r}_{10},t) = \boldsymbol{E}_{\mathrm{ext}}(\boldsymbol{r}_{10},t) + \boldsymbol{E}_{r2}(\boldsymbol{r}_{10},t). \tag{3.13}$$

The substitution of (3.3) and (3.8)-(3.10) into (3.12) gives

$$F_1 = \frac{e_1^2 e_2^2 (k + kn)(E_0^2 - (E_0 \cdot n)^2)}{2m_1 m_2 \omega^2 c^2 l} \sin(kl + k \cdot l). \tag{3.14}$$

Analogously

$$F_2 = \frac{e_1^2 e_2^2 (k - kn)(E_0^2 - (E_0 \cdot n)^2)}{2m_1 m_2 \omega^2 c^2 l} \sin(kl - k \cdot l). \tag{3.15}$$

These expressions can be represented as $F_1 = \partial U_1/\partial l$ and $F_2 = -\partial U_2/\partial l$, where

$$U_1 = -\frac{e_1^2 e_2^2 (E_0^2 - (E_0 \cdot n)^2)}{2m_2 m_2 \omega^2 c^2 l} \cos(kl + k \cdot l) \tag{3.16}$$

$$U_2 = -\frac{e_1^2 e_2^2 (E_0^2 - (E_0 \cdot n)^2)}{2m_2 m_2 \omega^2 c^2 l} \cos(kl - k \cdot l). \tag{3.17}$$

The quantity U_1 (U_2) is the energy of the first (second) particle in the field created by the second (first) particle. This energy is inversely proportional to the distance between the charges, i.e. it behaves like the Coulomb energy, but has an oscillating factor. However, for the comparison of the forces, the gradients of the energies are important, not the energies themselves. Therefore at large distance the radiative force may be dominant over Coulomb force. This leads to the principle possibility of forming bound states between even two like charges. The comparison between the radiative interaction force and the Coulomb force gives the value of the electric intensity, E_0, for which these forces are equal, namely $E_0 \simeq m\omega c e^{-1}/\sqrt{kl}$. At wavelength $\lambda = 1$ mu and $kl = 10$ for electrons, one obtains for corresponding energy current $W = cE_0^2/4\pi \approx 0.23 \times 10^{18}$ w/cm^2.

Let us compare the radiative force of the interaction with the average force acting on the charged particle from the incident electromagnetic wave [19]

$$f_{1,2} = \frac{8\pi}{3} \left(\frac{e_{1,2}^2}{m_{1,2} c^2} \right) \frac{\langle E_{\text{ext}}^2 \rangle}{4\pi k} k$$

analogous to the force of radiative pressure [14, 17, 18]. For particles with approximately equal masses and charges, the equations (3.14) and (3.15) result in $|F|/|f| \sim (kl)^{-1} \ll 1$. If the particles are equal then $f_1 = f_2$ and they move together.

Thus there is some identity between the radiative interaction of charged particles under the action of the electromagnetic field and the interaction of the small particles in a liquid under the action of sound waves.

4. The radiative long-range forces between two interacting particles in a classical field

The Lagrangian of such a system is given by [20]

$$\mathcal{L} = \frac{m_2 \dot{r}_1^2}{2} + \frac{m_2 \dot{r}_2^2}{2} + e_1 \varphi(r_1, t) + e_2 \varphi(r_2, t) + \frac{1}{2} \int \mathrm{d}V \left(\frac{\dot{\varphi}^2}{c^2} - (\nabla \varphi)^2 \right)$$

where $m_{1,2}$ are the masses of the particles and $e_{1,2}$ are the coupling constants. The canonical equations are

$$\begin{aligned} &m_{1,2} \ddot{r}_{1,2} = e_{1,2} \nabla \varphi(r_{1,2}, t) \\ &\frac{1}{c^2} \ddot{\varphi}(r, t) - \Delta \varphi(r, t) = e_1 \delta(r - r_1(t)) + e_2 \delta(r - r_2(t)). \end{aligned} \tag{4.1}$$

We suppose that the external scalar wave is $\varphi = A\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$. Let $\boldsymbol{r}_{10}$ and $\boldsymbol{r}_{20}$ be the position vectors of the mean positions of the two particles and $\boldsymbol{\xi}_{1,2}$ be their displacements.

It follows from the expressions (4.1) that the scalar potential created by the dipole oscillations of the particle is described by the same equation as the scalar potential of the electromagnetic field. Consequently we use the well known expressions for the delayed electromagnetic potentials [19]

$$\varphi_{1,2} = \frac{e_{1,2}}{4\pi(R_{1,2} - \dot{\boldsymbol{\xi}}_{1,2}\cdot\boldsymbol{R}_{1,2}/c)} \approx \frac{e_{1,2}}{4\pi R_{1,2}}(1 + \dot{\boldsymbol{\xi}}_{1,2}\cdot\boldsymbol{n}_{1,2}/c). \tag{4.2}$$

The remainder of the calculation is analogous to those performed previously. Therefore we just write the final result for the long-range radiative forces

$$\boldsymbol{F}_1 = \frac{e_1^2 e_2^2 A^2(\boldsymbol{n}\cdot\boldsymbol{k})^2(\boldsymbol{k} + k\boldsymbol{n})\sin(kl + \boldsymbol{k}\cdot\boldsymbol{l})}{8\pi m_1 m_2 \omega^2 c^2 l} = \frac{\partial U_1}{\partial \boldsymbol{l}} \tag{4.3}$$

$$\boldsymbol{F}_2 = \frac{e_1^2 e_2^2 A^2(\boldsymbol{n}\cdot\boldsymbol{k})^2(\boldsymbol{k} - k\boldsymbol{n})\sin(kl - \boldsymbol{k}\cdot\boldsymbol{l})}{8\pi m_1 m_2 \omega^2 c^2 l} = -\frac{\partial U_2}{\partial \boldsymbol{l}} \tag{4.4}$$

where

$$U_1 = -\frac{e_1^2 e_2^2 A^2(\boldsymbol{n}\cdot\boldsymbol{k})^2\cos(kl + \boldsymbol{k}\cdot\boldsymbol{l})}{8\pi m_1 m_2 \omega^2 c^2 l}$$

$$U_2 = -\frac{e_1^2 e_2^2 A^2(\boldsymbol{n}\cdot\boldsymbol{k})^2\cos(kl - \boldsymbol{k}\cdot\boldsymbol{l})}{8\pi m_1 m_2 \omega^2 c^2 l}.$$

It can be seen that the structure of the forces (4.3) and (4.4) is analogous to the radiative forces (3.14) and (3.15).

5. Conclusions

If the radiative transfer is carried out by the massless classical field (acoustic, electromagnetic, etc) then the induced oscillations of the small particles result in the appearance of long-range radiative forces. These forces are of two types: (i) mutual attraction or repulsion of particles; (ii) transfer and mutual rotation. The time-averaged forces are proportional to the square of the field amplitude and inversely proportional to the distance between the particles.

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